DEVELOPMENT OF AN AUTOMATED STORAGE/RETRIEVAL SYSTEM

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ABSTRACT: This paper shows the mathematical model of an automated storage/retrieval system (AS/RS) based on initial condition. We iditificate oscillation modes and kinematics displacement of system on the basis model results. With the use of the present model, the automated warehouse cranes system can be design more efficiently. Also, a AS/RS model with the control system are implemented to show the effectiveness of the solution. This research is part of R/D research project of HCML Department of Science and Technology to meet the demand of the manufacturing of automated warehouse in VIKYNO corporation, in particular, and in VietNam corporations, in general.

Keywords: automated storage/retrieval system, AS/RS model

1. INTRODUCTION

An AS/RS is a robotic material handling system (MHS) that can pick and deliver material in a direct-access fashion. The selection of a material handling system for a given manufacturing system is often an important task of mass production in industry. One must carefully define the manufacturing environment, including nature of the product, manufacturing process, production volume, operation types, duration of work time, work station characteristics, and working conditions in the manufacturing facility.

Hence, manufacturers have to consider several specifications: high throughput capacity, high IN/OUT rate, high reliability and better control of inventory, improved safety condition, saving investment costs, managing professionally and efficiently. This system has been used to supervise and control for automated delivery and picking [1], [2], [3].

In this paper, several design hypothesis is given to propose a mathematical model and emulate to iditificate oscillation modes and kinematic displacement of system based on initial conditions of force of load. As a result, we decrease error and testing effort before manufacturing [4], [5]. No existing AS/RS met all the requirements. Instead of purchasing an existing AS/RS, we chose to design a system for our need of study period and present manufacturing in VietNam.

This works was implemented at Robotics Division, National Laboratory of Digital Control and System Engineering (DCSELAB).
2. MODELLING OF AS/RS

An AS/RS is a robot that composed of (1) a carriage that moves along a linear track (x-axis), (2) one/two mast placed on the carriage, (3) a table that moves up and down along the mast (y-axis) and (4) a shuttle-picking device that can extend its length in both direction is put on the table. The motion of picking/placing an object by the shuttle-picking device is performed horizontally on the z-axis.

In this paper, an AS/RS is considered a none angular deflection construction in cross section in place where having concentrated mass [4], [5], [6]. There are several assumtions as follows:

The weight of construction post is concentrated mass in floor level (Fig. 1).

Structural deformation is not depend on bar axial force. Assume that the mass of each part in AS/RS is given as  \( m_1, m_2, m_3 \), and \( m_L \) is lifting mass.

When operation, there are two main motions: translating in horizontal direction with load \( f_1 \); translating in vertical direction with load \( f_2 \). The innitial conditions of AS/RS are lifting mass, lifting speed, lifting height, moving speeds, inertia force, resistance force, which can be used to establish mathematical model of AS/RS and verify the system behavior.

The assumed parameters of the AS/RS are given in Table 1.

![Fig. 1: Model of AS/RS](image-url)
2.1 Mathematical Model

Case 1: Horizontal moving along steel rail with load $f_1$ [7]

It is assumed that (1) Structural deformation is not dependent on bar axial force; (2) The mass in each part of automated warehouse cranes is given as $m_1$, $m_2$, $m_3$, in the case, $m_L$ is lifting mass; (3) When the system moves, there are two main motions: travelling along steel rail under load $f_1$ and lifting body vertical direction under load $f_2$.

The following model for traveling can be obtained:

$$m_1\ddot{x}_1 + k_1(x_1 - x_2) + C_1\dot{x}_1 = f_1$$

$$m_2\ddot{x}_2 + k_1(x_2 - x_1) + k_2(x_2 - x_3) = 0$$

$$m_3\ddot{x}_3 + k_2(x_3 - x_2) + C_2\dot{x}_3 = 0$$

where $m_{13} = m_1 + m_2 + m_L + m_3$

$m_{23} = m_2 + m_L + m_3$

$$k_1 = \frac{6EI}{x_4^3}, \quad k_2 = \frac{6EI}{(L - x_4)^3}$$

with $x_4 = \frac{L}{2}$, then $k_1 = k_2$

where $E$: elastic coefficient of material

$I$: second moment of area

$k_1, k_2$: stiffness proportionality

Case 2: Vertical moving with load $f_2$ [8]

$$m_2L\ddot{x}_4 + k_c x_4 + C_4\dot{x}_4 = f_2$$

where $m_{2L} = m_2 + m_L$

$$k_c$$: stiffness of cable

$$k_c = \frac{4E}{D^2} = \frac{4E}{(L-x_4)^2}$$

$D$: diameter of cable

2.2 Solution of motion equation

a. Travelling along steel rail underload $f_1$
If resistance force is skipped, the motion equation can be written as:

\[
\begin{bmatrix}
    m_{13} & 0 & 0 \\
    0 & m_{23} & 0 \\
    0 & 0 & m_3
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_1 \\
    \ddot{x}_2 \\
    \ddot{x}_3
\end{bmatrix}
+
\begin{bmatrix}
    k_1 & -k_1 & 0 \\
    -k_1 & k_1 + k_2 & -k_2 \\
    0 & -k_2 & k_2
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3
\end{bmatrix}
=
\begin{bmatrix}
    f_1 \\
    0 \\
    0
\end{bmatrix}
\]  

or in the matrix form \(M \ddot{x} + kx = F\)

Solution of Eq. (1) can be solved by superposition method [9] as the followings:

Eigen problem: \(k \phi = M \phi \omega^2 \Rightarrow (k - M \omega^2) \phi = 0\)

where \(\phi\) : n level vector
\(\omega\) : vibration frequency (rad/s).

\[
\begin{vmatrix}
    k_1 + m_{13} \omega^2 & -k_1 & 0 \\
    -k_1 & k_1 + k_2 + m_{23} \omega^2 & -k_2 \\
    0 & -k_2 & k_2 - m_3 \omega^2
\end{vmatrix}
= 0
\]

At the position \(x_4 = \frac{L}{2}\)

Substituting constant values in Table 1 into Eq. (3), we have

\[
a \phi_1 \phi_2 \phi_3 = a \phi_1 \phi_2 = a \phi_2^2 \Rightarrow a = \pm 0.025
\]

\[
-b \phi_3^T k \phi_3 = \omega_3^2
\]

\[
b \phi_1 \phi_2 \phi_3 = b \phi_1 \phi_2 = b \phi_2^2 \Rightarrow b = \pm 1.183 \times 10^{-3}
\]

If \(a\) and \(b\) are positive, \(\phi_2\) values is as follows:

\[
\phi_2 = [0.025 \ -0.031 \ -0.033]^T, \quad \phi_3 = [0.001 \ -0.041 \ 0.181]^T
\]
It can be seen that the condition $\phi^TM\phi = I$ is satisfied.

If resistance force is skipped, the motion equation will be written as the followings:

$$\mathbf{M}\ddot{x}(t) + \mathbf{C}\dot{x}(t) + \mathbf{K}x(t) = \mathbf{f}(t)$$

and n individual equation can be written:

$$\ddot{x}_i(t) + \omega_i^2 x_i(t) = R_i(t)$$

where:

$$x_i(t) = \frac{1}{\omega_i} \int R_i(t) \sin \omega_i (t - \tau) d\tau + \alpha_i \sin \omega_i t + \beta_i \cos \omega_i t$$

(6)

$$\ddot{x}_i(t) = \frac{R_i(t)}{\omega_i} \sin \omega_i t - \alpha_i \omega_i \cos \omega_i t - \beta_i \omega_i \sin \omega_i t$$

(7)

$x_i(0) = x_i$,

$\dot{x}_i(0) = \dot{x}_i$

$\omega_i$ and $\beta_i$ can be specified from initial conditions

$$x_i(0) = \phi_i^T M \phi_i u$$

$$\ddot{x}_i(0) = \phi_i^T M \ddot{\phi}_i$$

Geometric inversion can be defined by principle of superposition:

$$u(t) = \{x(t)\} - \sum_{j=1}^{n} \{x_j(t)\} + \{x_{n+1}(t)\}$$

Displacement of point is defined by principle of superposition [9]

$$u_j(t) = \sum_{j} \phi_j x_j(t)$$

If resistance forces are considered

Using integral Duhamel to find motion equation [9]:

$$x_i(t) = \frac{1}{\omega_i} \int R_i(t) e^{-\zeta_i \omega_i (t-\tau)} \sin \omega_i (t-\tau) d\tau + e^{-\zeta_i \omega_i t} \left( a_i \sin \omega_i t + \beta_i \cos \omega_i t \right)$$

(9)

where:

$$\zeta_i = \frac{\omega_i}{2}$$

$$\alpha_i = \zeta_i \omega_i$$

$$\beta_i = \omega_i$$

We find $\alpha_i$ and $\beta_i$ value based on initial condition

Displacement of point is defined by principle of superposition (Eq. (8))

Influential dynamic load act on warehouse cranes in some cases

The acting force is a constant and system has influential resistance force

It is assumed that $f_i = W_i = 423.6$ (N)

From Eq. (4) and Eq. (5), we have

$R_1(t) = 0.025 f_1(t)$

$R_2(t) = 0.001 f_1(t)$

From Eq. (10)
Substituting the values: \( \omega_2 \), \( \xi_2 \) into Eq. (9), and from initial condition:

\[
\begin{bmatrix}
\dot{x}_2 \\
\ddot{x}_2
\end{bmatrix}
\bigg|_{t=0} = \begin{bmatrix}
\phi_2 T M^R \\
0
\end{bmatrix} = 0
\]

From Eq. (6) and Eq. (7) with \( \alpha_2 = 0 \),

\[
\beta_2 = 0
\]

\[
x_2(t) = 9.42 \left( 1 - e^{-0.02t} \right) \left( \cos 1.06t + 0.02 \sin 1.06t \right)
\]

with \( \omega_3 = 4.25 \):

\[
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix} =
\begin{bmatrix}
0 \\
9.42 \left( 1 - e^{-0.02t} \right) \left( \cos 1.06t + 0.02 \sin 1.06t \right) \\
0.023 \left( 1 - e^{-0.085t} \right) \left( 1 - e^{-0.085t} \left( \cos 4.25t + 0.02 \sin 4.25t \right) \right)
\end{bmatrix}
\]

(12)

With the force of load is periodic, resistance force of the system is assumed to be \( f_1 = A \cos \omega t \)

From Eq. (4) and (5), we have

\[
R_2(r) = 0.025 f_1 = 0.025 A \cos \omega t
\]

\[
R_3(r) = 0.001 f_1 = 0.001 A \cos \omega t
\]

Solution \( x_2(t) \)

Substituting \( R_2(r), \xi_2, \xi_2 \) into Eq. (9) and initial condition into Eq. (9) and Eq. (11), we have

\[
x_2(t) = 22.24 \times 10^{-3} \ A \cos \omega t \left( 1 - e^{-0.02t} \right) \left( 1 - e^{-0.085t} \left( \cos 1.06t + 0.02 \sin 1.06t \right) \right)
\]

Solution \( x_3(t) \)

Substituting \( R_3(r), \xi_3, \xi_3 \) into Eq. (9) and initial condition into Eq. (9) and Eq. (11), we have

\[
x_3(t) = 5.534 \times 10^{-5} \ A \cos \omega t \left( 1 - e^{-0.085t} \right) \left( \cos 4.25t + 0.02 \sin 4.25t \right)
\]
The motion equation under periodic load can be derived as follows:

\[
\begin{align*}
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix} &= 
\begin{bmatrix}
0 \\
22.24 \times 10^{-7} \cos \omega t \\
5.534 \times 10^{-7} \cos \omega t
\end{bmatrix} \\
+ 
\begin{bmatrix}
(1-\mathrm{e}^{-0.02(\cos 1.06 t + \sin 1.06 t)}) \\
(1-\mathrm{e}^{-0.085(\cos 4.25 t + \sin 4.25 t)}) \\
(1-\mathrm{e}^{-0.085(\cos 4.25 t + \sin 4.25 t)}) 
\end{bmatrix} \\
\begin{bmatrix}
0 \\
0.023 \cos 40 t \\
0.023 \cos 40 t
\end{bmatrix}
\end{align*}
\]  
(13)

It is assumed that \( f_1 = 423.6 \cos 40t \)

Equation (13) becomes

\[
\begin{align*}
\begin{bmatrix}
x_1(0) \\
x_2(0) \\
x_3(0)
\end{bmatrix} &= 
\begin{bmatrix}
0 \\
0.42 \cos 40 t(1-\mathrm{e}^{-0.02(\cos 1.06 t + \sin 1.06 t)}) \\
0.023 \cos 40 t(1-\mathrm{e}^{-0.085(\cos 4.25 t + \sin 4.25 t)})
\end{bmatrix}
\end{align*}
\]  
(14)

b. Lifting carrier in vertical direction under load \( f_2 \)

The model of lifting carrier can be written as

\[
m_2 L \ddot{x} + \ddot{c} \varrho \dot{x} + c_4 x = f_2
\]  
(15)

Skipping resistance force and \( f_2 = 0 \), we have

\[
\dot{x}(t) = \alpha_4 \sin \omega_4 t + \beta_4 \cos \omega_4 t
\]  
(16)

\[
\ddot{x}(t) = \alpha_4 \omega_4 \cos \omega_4 t - \beta_4 \omega_4 \sin \omega_4 t
\]  
(17)

where \( \omega_4 = \frac{k_c}{m_2 L} = 6594 \div 550 = 11.989 \text{ (rad/s)} \)

\[
\alpha_4, \beta_4 \text{ are defined from initial condition.}
\]

when \( t = 0 \): \( x_4 = 10 \text{ (m)}, \varrho_4 = 1 \text{ (m/s)}. \)

Substituting the values into Eq. (16), Eq. (17), we have \( \beta_4 = 10, \alpha_4 = 0.083 \)

Oscillation system is of a harmonic motion as

\[
x_4(t) = 0.083 \sin 11.989 t + 10 \cos 11.989 t
\]

If resistance forces are considered, using integral Duhamel to find the motion equation

\[
x_4(t) = \frac{1}{\varrho_4} \int_0^t R_4(\tau) \mathrm{e}^{-\varrho_4(t-\tau)} \sin \varrho_4(t-\tau) \mathrm{d}\tau + \mathrm{e}^{-\varrho_4 t} \left( \alpha_4 \sin \varrho_4 t + \beta_4 \cos \varrho_4 t \right)
\]  
(19)

where \( \varrho_4 = \varrho_4 \sqrt{1 - \varrho_4^2}, \varrho_4 = 11.989 \sqrt{1 - 0.02^2} = 11.987 \text{ (rad/s)} \)

\( \alpha_4, \beta_4 \) can be derived from the initial condition.

\[
x_4(t) = \frac{R_4(t)}{\varrho_4^2 + \varrho_4^2 \omega_4^2} \left( 1 - \mathrm{e}^{-\varrho_4 t} \left( \cos \omega_4 t + \frac{\varrho_4}{\omega_4} \sin \omega_4 t \right) \right) + \mathrm{e}^{-\varrho_4 t} \left( \alpha_4 \sin \omega_4 t + \beta_4 \cos \omega_4 t \right)
\]  
(20)
\[ R(\tau) = \frac{R_1(t)}{\omega_4^2 + \xi_4 \omega_4^2} \left( \frac{\xi_4 \omega_4^2}{\omega_4^2} + \frac{\xi_4}{\omega_4^2} \right) \sin \omega_4 t - \]

\[ e^{-\xi_4 \omega_4 t} \left( (\xi_4 \omega_4 a_4 + \beta_4 \omega_4) \sin \omega_4 t + (\xi_4 \omega_4 \beta_4 - a_4 \omega_4) \cos \omega_4 t \right) \]

when \( t = 0: x_4 = 10 \text{ (m)}, \ \dot{x}_4 = 1 \text{ (m/s)}. \)

Substituting above values into Eq. (20), we have \( \beta_4 = 10. \)

Substituting above values into Eq. (21), we have 
\[ 1 = \left( \xi_4 \omega_4 \beta_4 - a_4 \omega_4 \right) \]

\[ \Rightarrow a_4 = \frac{\xi_4 \omega_4 \beta_4 - 1}{\omega_4} = \frac{0.02 \times 11.989 - 1}{11.987} = -63.4 \times 10^{-4} \]

Substituting \( \alpha_4, \beta_4, a_4, \omega_4 \) into Eq. (20), we have

\[ x_4(t) = \frac{R_1(t)}{143.75} \left( 1 - e^{0.24t} \left( \cos 11.987t + 0.02 \sin 11.987t \right) + e^{0.24t} \left( -64.3 \times 10^{-4} \sin 11.987t + 10 \cos 11.987t \right) \right) \] (22)

With the force of load is a costant, the resistance force is assumed to be \( f_2 = S_{\text{max}} = 1736.76 \text{ N} \).

Substituting \( R_1(t) = f_2 = 1736.76 \text{ (N)} \) into Eq. (22):

\[ x_4(t) = 12.08 \left( 1 - e^{0.24t} \left( \cos 11.987t + 0.02 \sin 11.987t \right) + e^{0.24t} \left( -64.3 \times 10^{-4} \sin 11.987t + 10 \cos 11.987t \right) \right) \]

Or

\[ x_4(t) = 12.08 \left( 1 - e^{0.24t} \left( 0.172 \cos 11.987t + 0.02 \sin 11.987t \right) \right) \] (23)

The force of load is periodic, the resistance force of the system is assumed to be

\[ f_2 = 1736.76 \cos 40t \]

Substituting \( R_1(t) = f_2 = 1736.76 \cos 40t \) into Eq. (22):

\[ x_4(t) = \frac{1736.76 \cos 40t}{143.75} \left( 1 - e^{0.24t} \left( \cos 11.987t + 0.02 \sin 11.987t \right) + e^{0.24t} \left( -64.3 \times 10^{-4} \sin 11.987t + 10 \cos 11.987t \right) \right) \]

Or

\[ x_4(t) = 12.08 \cos 40t \left( 1 - e^{-0.24t} \left( 0.172 \cos 11.987t + 0.02 \sin 11.987t \right) \right) \] (24)

### 2.3 Simulation Results

**a. The carrier travelling along rail under load** \( f_1 \)

From the motion equation, the system can be simulated to describe the oscillation and displacement of the robot on time and use Eq. (10) to define displacement of point [10].

Trang 32
The force of load is constant, the resistance force is assumed to be $f_1 = 423.6$ (N)

From Eq. (12), the system motion is as follows:

$$
\begin{align*}
\{x_1(t) &= 0 \\
x_2(t) &= 9.42 \left(1-e^{-0.06t} \left(\cos1.06t+0.02\sin1.06t\right)\right) \\
x_3(t) &= 0.023 \left(1-e^{-0.085t} \left(\cos4.25t+0.02\sin4.25t\right)\right)
\end{align*}
$$

$x_1(t) = 0$, the plot $x_2(t)$ and $x_3(t)$ is shown in Fig. 2.

Fig. 2. System oscillation under constant with $\omega = 1.06$ (rad/s)

Fig. 3. System oscillation under constant load with $\omega = 4.25$ (rad/s)
The point's displacement is defined by the principle of superposition.

From Eq. (8), we have

\[
\begin{align*}
\mathbf{u}(t) &= \begin{pmatrix}
0.24 \left(1 - e^{-0.02t} \cos1.06t + 0.02\sin1.06t\right) + \\
0.23 \times 10^{-4} \left(1 - e^{-0.085t} \cos4.25t + 0.02\sin4.25t\right) \\
-0.29 \left(1 - e^{-0.02t} \cos1.06t + 0.02\sin1.06t\right) - \\
9.43 \times 10^{-4} \left(1 - e^{-0.085t} \cos4.25t + 0.02\sin4.25t\right) \\
-0.3 \left(1 - e^{-0.02t} \cos1.06t + 0.02\sin1.06t\right) + \\
4.163 \times 10^{-4} \left(1 - e^{-0.085t} \cos4.25t + 0.02\sin4.25t\right)
\end{pmatrix} \\
\end{align*}
\]  

(25)

The point's displacements are given in Table 2.

**Table 2.** The displacement of points

<table>
<thead>
<tr>
<th>Time</th>
<th>Displacement $u_1$ (m)</th>
<th>Displacement $u_2$ (m)</th>
<th>Displacement $u_3$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>$4.8178 \times 10^{-5}$</td>
<td>$-6.1618 \times 10^{-5}$</td>
<td>$-4.5204 \times 10^{-5}$</td>
</tr>
<tr>
<td>0.04</td>
<td>$2.0415 \times 10^{-4}$</td>
<td>$-2.602 \times 10^{-4}$</td>
<td>$-1.952 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.06</td>
<td>$4.6777 \times 10^{-4}$</td>
<td>$-5.956 \times 10^{-4}$</td>
<td>$-4.505 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.08</td>
<td>$8.3882 \times 10^{-4}$</td>
<td>0.0011</td>
<td>8.112 $\times 10^{-4}$</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0013</td>
<td>0.0017</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

**Table 3.** Point Displacement

<table>
<thead>
<tr>
<th>Time</th>
<th>Displacement $u_1$ (m)</th>
<th>Displacement $u_2$ (m)</th>
<th>Displacement $u_3$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>$3.3624 \times 10^{-5}$</td>
<td>$-4.3007 \times 10^{-5}$</td>
<td>$-3.2709 \times 10^{-5}$</td>
</tr>
<tr>
<td>0.04</td>
<td>$5.971 \times 10^{-6}$</td>
<td>7.6123 $\times 10^{-6}$</td>
<td>5.9202 $\times 10^{-6}$</td>
</tr>
<tr>
<td>0.06</td>
<td>$3.455 \times 10^{-4}$</td>
<td>4.3995 $\times 10^{-4}$</td>
<td>3.4483 $\times 10^{-4}$</td>
</tr>
<tr>
<td>0.08</td>
<td>$8.387 \times 10^{-4}$</td>
<td>0.0011</td>
<td>8.4055 $\times 10^{-4}$</td>
</tr>
<tr>
<td>0.1</td>
<td>$8.622 \times 10^{-4}$</td>
<td>0.0011</td>
<td>8.6695 $\times 10^{-4}$</td>
</tr>
</tbody>
</table>
With the force of load is periodic, resistance force of the system is assumed to be 
\[ f_1 = 423.6 \cos 40t \]

From Eq. (18), we have
\[
\begin{align*}
\begin{bmatrix}
    x_1(t) \\
x_2(t) \\
x_3(t)
\end{bmatrix} &=
\begin{bmatrix}
    0 \\
    9.42 \cos 40t (1 - e^{-0.02t} \cos 1.06t + 0.02 \sin 1.06t) \\
    0.023 \cos 40t (1 - e^{-0.085t} \cos 4.25t - 0.02 \sin 4.25t)
\end{bmatrix}
\end{align*}
\]

The oscillation plot of \(x_2(t)\) and \(x_3(t)\) are described in Fig. 4 and Fig. 5.

Fig. 4. System oscillation under periodic load with \(\omega = 1.06\) (rad/s)
The point’s displacement is defined by principle of superposition.

From Eq. (8), we have

\[
\begin{align*}
\vec{u}_1(t) &= \begin{pmatrix}
0.24 \cos 40t \left(1 - e^{-0.02t} \left(\cos 1.06t + 0.02 \sin 1.06t\right)\right) + \\
0.23 \times 10^{-4} \cos 40t \left(1 - e^{-0.085t} \left(\cos 4.25t + 0.02 \sin 4.25t\right)\right) \\
-0.29 \cos 40t \left(1 - e^{-0.02t} \left(\cos 1.06t + 0.02 \sin 1.06t\right)\right)
\end{pmatrix} \\
\vec{u}_2(t) &= \begin{pmatrix}
9.74 \times 10^{-4} \cos 40t \left(1 - e^{-0.085t} \left(\cos 4.25t + 0.02 \sin 4.25t\right)\right) \\
-0.31 \cos 40t \left(1 - e^{-0.02t} \left(\cos 1.06t + 0.02 \sin 1.06t\right)\right)
\end{pmatrix} \\
\vec{u}_3(t) &= \begin{pmatrix}
42.36 \times 10^{-4} \cos 40t \left(1 - e^{-0.085t} \left(\cos 4.25t + 0.02 \sin 4.25t\right)\right)
\end{pmatrix}
\end{align*}
\]

(26)

The point displacement are given in Table 3.

b. Lifting the table in vertical direction under load \( f_2 \)

Resistance force is skipped and \( f_2 = 0 \). From Eq. (18), the oscillation system is of harmonic motion:

\[ x_4(t) = 0.083 \sin 11.989t + 10 \cos 11.989t \]
Fig. 6. Harmonic motion of system with $\omega = 11.989$ (rad/s)

Fig. 7. System oscillation under constant load with $\omega = 11.978$ (rad/s)

With the force of load is constant, the resistance force is assumed to be $f_2 = S_{max} = 1736.76$ N.

From Eq. (23), we have

$$x_1(t) = 12.08 \left( 1 - e^{-0.24t} \right) \left( 0.172 \cos 11.987t + 0.02 \sin 11.987t \right)$$

With the force of load is periodic, the resistance force of the system is assumed to be

$$f_2 = 1736.76 \cos 40t$$
From Eq. (24), we have $x_A(t)=12.08\cos 40t \left(1-e^{-0.24t} \left(0.172\cos 11.987t+0.02\sin 11.987t\right)\right)$

![Graph of system oscillation under periodic load with $\omega = 11.987$ (rad/s)](image)

Fig. 8. System oscillation under periodic load with $\omega = 11.987$ (rad/s)

From the above plots, it can be realized that if we change the vibration frequency or load, the system oscillation and displacement will be changed. Alternatively, the vibration frequency is dependent on the lifting body mass, lifting height, stiffness proportionality... Hence, if we change initial condition design, we will identify oscillation modes and kinematic displacement of the system.

3. CONTROL SYSTEM DEVELOPMENT

There are three computers used to implement the control logic throughout the factory: host computer, client computer, and station computer. The host computer’s function is managing the database of the system, the client computer’s function is handling in/out operations, and the station computer’s function is monitoring and controlling the AS/RS system. The control system architecture is designed to meet the demand of a AS/RS is shown in Fig. 9.
In other words, the control system is composed of two control levels: management control and machine control. The communication between them is via LAN network. As for management control, a server host computer is installed with Warehouse Management software which connect to the warehouse database using Microsoft SQL Server framework. The server host can perform tasks, such as supplier management, customer management, items management, warehouse structure management. A barcode system is used for the item’s identification in warehouse. The interface of Warehouse Management software is shown in Fig. 10.

As for the machine control, a PAC 5010KW with SCADA system is implemented to control the motion of robot for in/out operations as shown in Fig. 11, and the control panel on AS/RS, Fig. 12. The design has allocated for VIKYNO company’s warehouse as shown in Fig. 13.
4. CONCLUSION

In this paper, mathematical model of the AS/RS is established with several oscillation modes and the kinematic displacement of the system are found respectively. The generalized calculate program was established to verify the behavior of the system and the system identification process. Finally, the development of this system have been done, but the experimental data yet to finish at this time of this writing. It is our works in the future.
PHÁT TRIỂN HỆ THỐNG LƯU KHO TỰ ĐỘNG

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Từ khóa: AS/RS, Automated storage/retrieval system.

REFERENCE
